EEL 316 Minor Test II Semester II 2014-2015

Answer all questions (Q.1: 20 marks, Q.2: 20 marks) Full Marks: 40

1. A pulse $p(t)$ given by $p(t) = \dfrac{\mathrm{sinc}(2Bt)}{1 - 4B^2t^2}$, $B > 0$, has the spectrum $P(f)$ given by

$$P(f) = \frac{1}{2B}\left[1 - \sin\left(\frac{\pi(|f| - 0.5B)}{B}\right)\right] \mathrm{rect}\left(\frac{f}{2B}\right).$$

A binary PAM signal is generated using $p(t)$ as $y(t) = \sum_k a_k\, p(t - kT_b)$, where $a_k \in \{-1, 1\}$.

(a) Find the energy of $p(t)$. [4]

(b) Find B_{rms}, the r.m.s. bandwidth of $p(t)$, given by [6]

$$B_{rms} = \left[\frac{\int_{-\infty}^{\infty} f^2 |P(f)|^2 df}{\int_{-\infty}^{\infty} |P(f)|^2 df}\right]^{1/2}.$$

(c) The signal $y(t)$ is passed through a tapped delay line equalizer with impulse response

$$h_{eq}(t) = w_{-1}\,\delta(t + T_b) + w_0\,\delta(t) + w_1\,\delta(t - T_b).$$

When $T_b = 1/(12B)$, find the tap weights w_{-1}, w_0, w_1 for which $p_{eq}(t) = p(t) \star h_{eq}(t)$ approximately satisfies the Nyquist criterion. Assume $p_{eq}(0) = 1$. [8]

(d) What is the minimum value of T_b (in terms of B) for which an equalizer is not needed? [2]

2. Equicorrelated real-valued signals $y_1(t), y_2(t), y_3(t)$ over a signaling interval $[0, T)$, each having energy E, and satisfying

$$\int_0^T y_i(t) y_j(t) dt = \rho E \quad \text{for } i \neq j, \quad 0 \leq \rho < 1,$$

are converted to another set of signals $\{s_1(t), s_2(t), s_3(t)\}$ by the transformation

$$s_1(t) = y_1(t), \qquad s_i(t) = s_{i-1}(t) + (-1)^{i-1} y_i(t), \quad i = 2, 3.$$

(a) What is the dimension N of the signal space $\{s_1(t), s_2(t), s_3(t)\}$? Starting with $s_1(t)$, obtain an orthonormal basis $\{\phi_1(t), \ldots, \phi_N(t)\}$ for the signal space using Gram-Schmidt orthogonalization. [12]

(b) Find the vectors $\underline{s}_1$ and $\underline{s}_3$ for the orthonormal basis found in (a). [2+6]

Some Formulae

- $\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if } |t| \leq \frac{T}{2}, \\ 0 & \text{if } |t| > \frac{T}{2}, \end{cases}$ $\qquad \mathrm{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$
- Fourier Transform pairs: $\mathrm{rect}\left(\frac{t}{T}\right) \leftrightarrow T\,\mathrm{sinc}(fT)$, $\exp(j2\pi f_0 t) \leftrightarrow \delta(f - f_0)$, $G(t) \leftrightarrow g(-f)$
- $\int \theta^2 \cos\theta\, d\theta = \theta^2 \sin\theta + 2\theta\cos\theta - 2\sin\theta$
- Matrix inverse:

$$\begin{bmatrix} 1 & a & b \\ a & 1 & a \\ b & a & 1 \end{bmatrix}^{-1} = \frac{1}{(1-b)(1+b-2a^2)} \begin{bmatrix} (1-a^2) & -a(1-b) & (a^2-b) \\ -a(1-b) & (1-b^2) & -a(1-b) \\ (a^2-b) & -a(1-b) & (1-a^2) \end{bmatrix}$$